

Transfer analysis of a local surface dynamo

Jonathan Pietarila Graham, Robert Cameron, and
Manfred Schüssler

Max-Planck-Institut für Sonnensystemforschung (MPS)

SYNS 2.5 April 2, 2009

Definition of a dynamo?

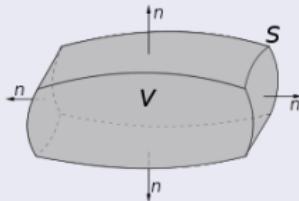
Formal statement of the kinematic dynamo problem (§6, Moffatt, 1978)

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{0} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

- $[\mathbf{B}]_S \equiv 0$ and $\mathbf{B} = O(r^{-3})$ as $r \rightarrow \infty$

$\mathbf{v}(\mathbf{x}, t)$ is a dynamo if $E_M(t) \not\rightarrow 0$ as $t \rightarrow \infty (\gg L^2/\eta)$

- $\mathbf{v} \cdot \mathbf{n} \equiv 0$ on boundary, S
- $E_K(t) \leq E_{max} \forall t$
- may be dynamo only for certain $\mathbf{B}(\mathbf{x}, 0)$ and η



Turbulence *is* capable of dynamo action

Turbulent dynamo

- Stretching of B-field lines by turbulence (Batchelor 1950)
- $$D_t \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{v} + \eta \nabla^2 \mathbf{B}$$

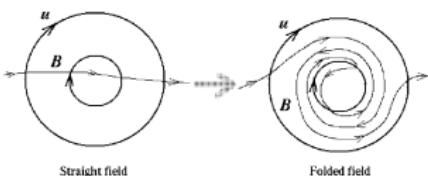
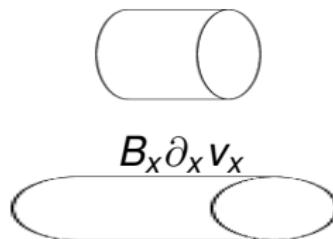


FIG. 2.—Stretching and folding of field lines by turbulent eddies.

Schekochihin et al. 2004



Overview of turbulent dynamo theory

Solar local surface dynamo – a small-scale dynamo?

How to tell – transfer analysis

What is a dynamo?

What type of dynamo is the local surface dynamo?

Turbulent dynamos – well studied

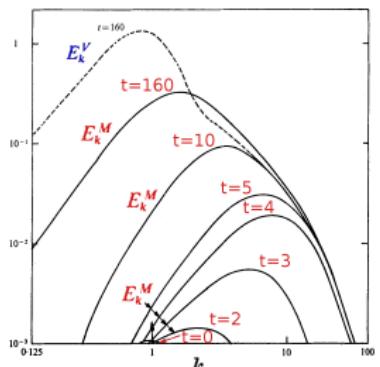


FIGURE 2. Growth to equilibrium of seed of M -energy. Nonhelicity. Only V -energy injection $P_V = P_0$ (defined in figure 1). Initial conditions: $E_k^V(0) = 6$ and $E_k^M(0) = 10^{-4}P_0$. Minimum and maximum wavenumbers: 2^{-4} and 2^7 . Magnetic Prandtl number unity, $\nu = \lambda = \frac{1}{32}$. Evolution of V - and M -energy spectra.

Pouquet et al. 1976

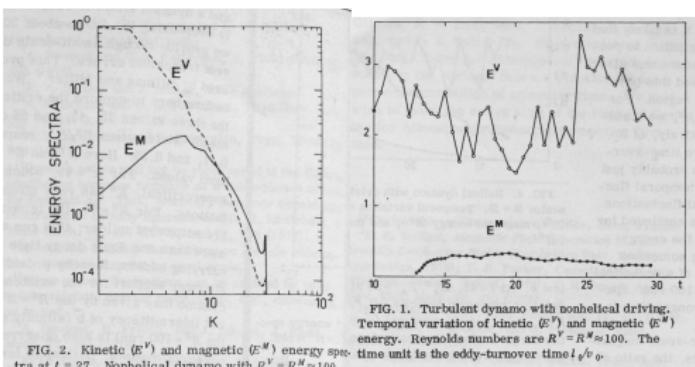


FIG. 1. Turbulent dynamo with nonhelical driving. Temporal variation of kinetic (E^V) and magnetic (E^M) energy. Reynolds numbers are $R^V = R^M \approx 100$. The time unit is the eddy-turnover time t_η/v_η .

Meneguzzi et al. 1981

Many demonstrations of kinematic small-scale dynamo (SSD)

Childress & Gilbert 1995; Maron & Cowley 2001; Heitsch et al. 2001; Schekochihin et al. 2004; Maron et al. 2004;

Ponty et al. 2005; Schekochihin et al. 2005; Mininni 2006; Ponty et al. 2007; Iskakov 2007, ...

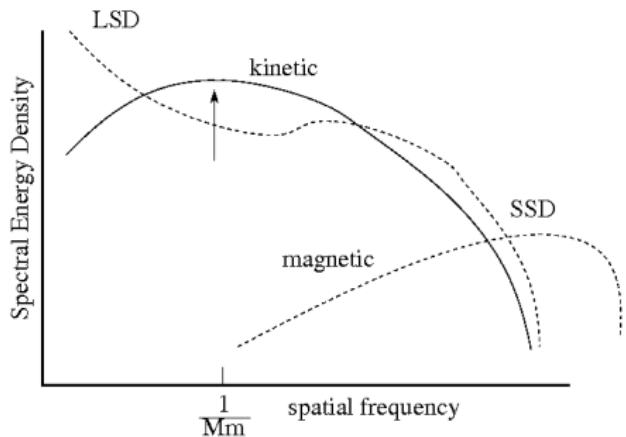
Turbulent dynamos – large & small

2 types of turbulent dynamos

- Large-scale (LSD; helicity, α -effect: mean-field)
- **Small-scale (SSD; non-helical)**
- $Re_M = \frac{\nu_o l_o}{\eta} > Re_M^C \rightarrow \text{dynamo}$

Solar surface convection can drive local dynamo action? $Re_M \sim 10^7$

- Petrovay & Szakaly 1993
- $\tau_{conv} \sim 10 \text{ min} \ll \tau_{rotation}$
 \rightarrow no net helicity

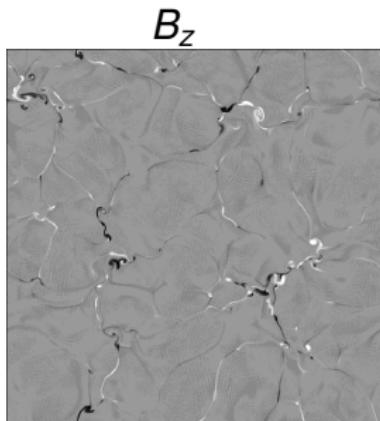
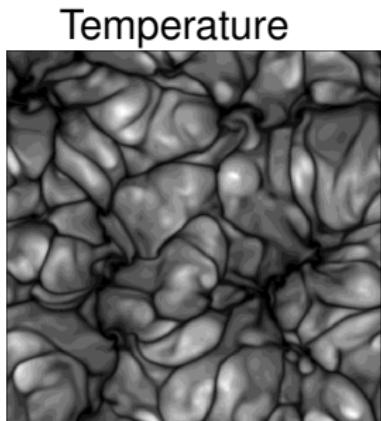


Frisch et al. 1975; Pouquet et al. 1976; Field et al. 1999;

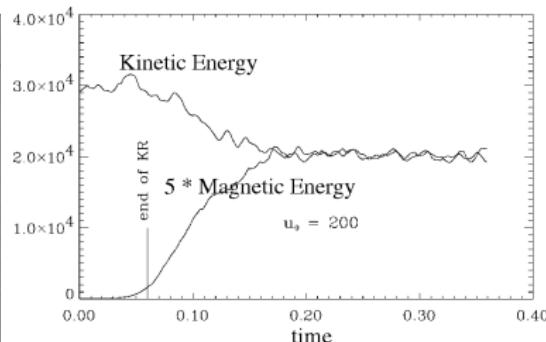
Brandenburg 2001; Brummell et al. 2001; Maron &

Blackman 2002; Maron et al. 2004; Mininni 2007; ...

Convectively-Driven Small-Scale Dynamo (Cattaneo 1999)



$$Re \approx 200$$
$$Re_M \approx 1000$$
$$512 \times 512 \times 97 \approx 5 \cdot Re_M^{9/4}$$



- Boussinesq convection (no sound waves) (see also Cattaneo et al. 2003)
- no Coriolis force (SSD)

Open questions

1. Strong stratification & little recirculation (Stein et al. 2003)

- Strong stratification
- Little plasma is recirculated in the near-surface layers
- Realistic magneto-convection with open boundaries
(Stein et al. 2003): $253 \times 253 \times 163 \rightarrow Re_M \sim 600$

NO DYNAMO

The MURaM code (Vögler et al. 2005; Vögler 2003)

Realistic magnetoconvection

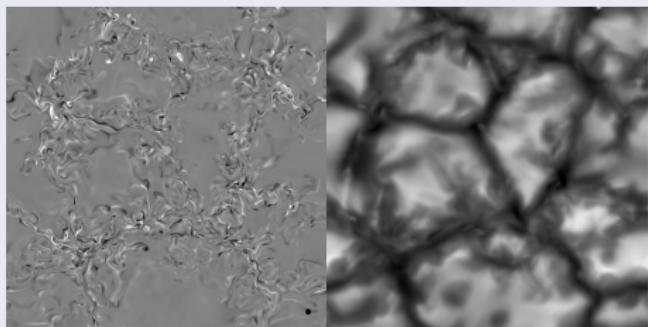
- Strong stratification
- Fully compressible
- Partial ionization
- Radiative transfer
- Open lower boundary

(vertical upflows, $\frac{dv}{dz} = 0$ for downflows; B_{par} not advected

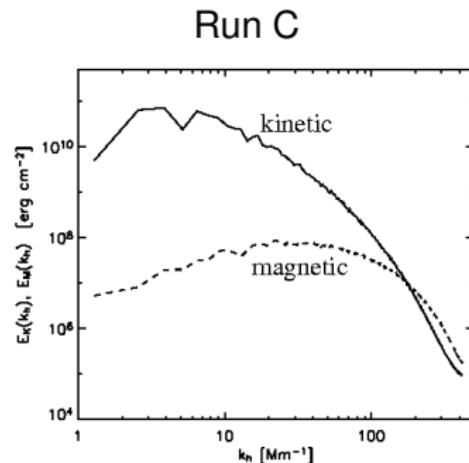
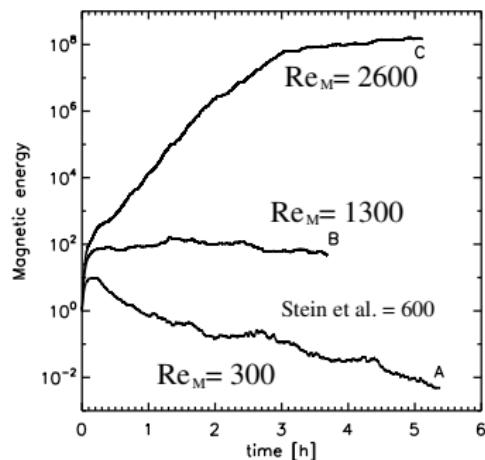
into box)

- No rotation
- Parallelized

B_z & brightness



Strong stratification & little recirculation – No Problem



(Vögler & Schüssler 2007)

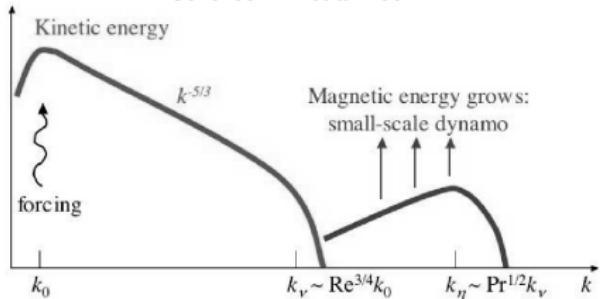
Open questions

$$2. Re \gg Re_M \text{ (i.e., } P_M \equiv \frac{Re_M}{Re} = \frac{\nu}{\eta} \ll 1)$$

- 1 Small eddies inhibit dynamo action? **NO** Ponty et al. 2005; Iskakov et al. 2007; Monchaux et al. 2007
 - 2 Small eddies + $B_0 \Rightarrow$ Alfvén waves
 - 3 * $B_0 +$ turbulence \Rightarrow cascade (*not* dynamo)

$$P_M > 1 \rightarrow l_\eta < l_\nu$$

Schekochihin et al. 2004



$$P_M < 1 \rightarrow l_\eta < l_\nu$$

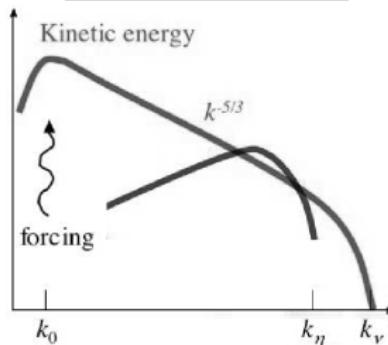
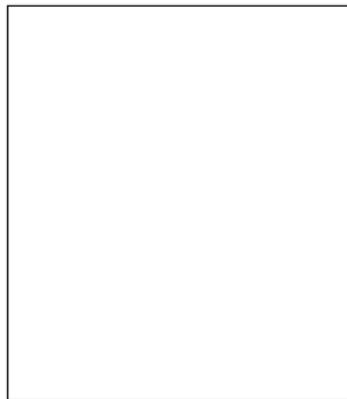


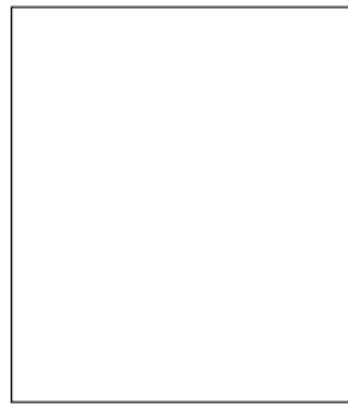
FIG. 1.—Sketch of scale ranges and energy spectra in a large- Pr_m medium.

How to tell what's going on – transfer analysis

Kinetic Energy



Magnetic Energy



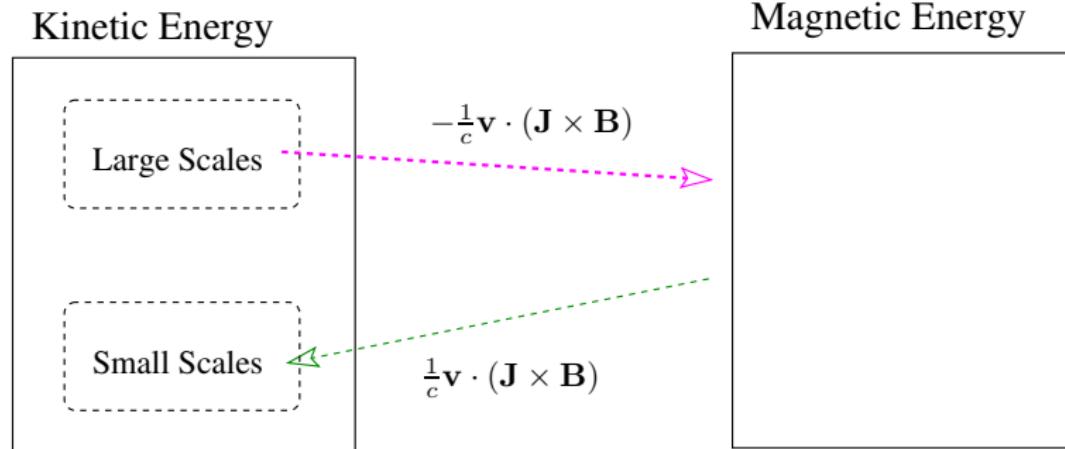
$$\frac{1}{c} \mathbf{v} \cdot (\mathbf{J} \times \mathbf{B})$$



$$\begin{aligned} & \frac{1}{4\pi} \mathbf{B} \cdot (\mathbf{B} \cdot \nabla \mathbf{v}) \\ & - \frac{1}{4\pi} |\mathbf{B}|^2 \nabla \cdot \mathbf{v} \end{aligned}$$



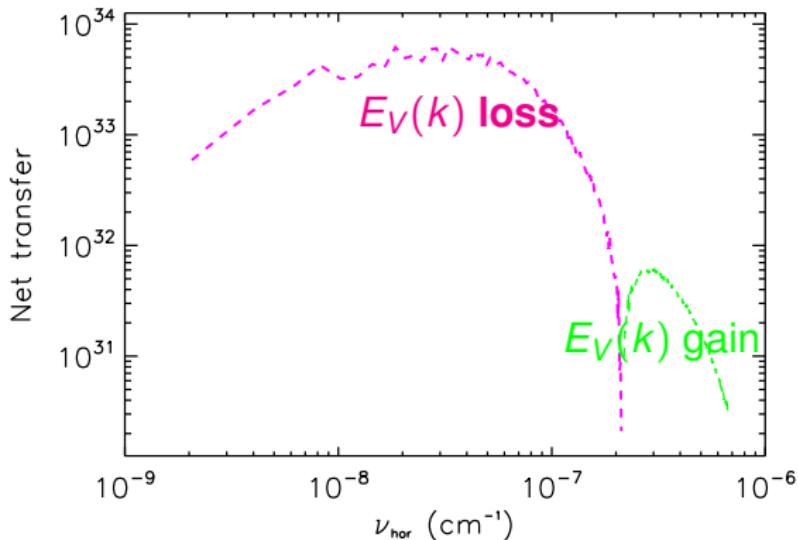
Which scales generate or receive magnetic energy?



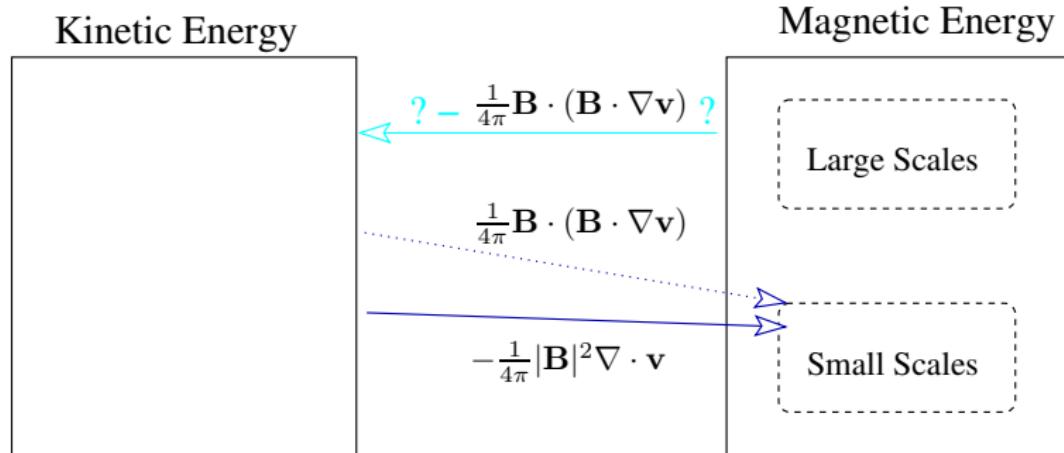
$$\sum_k T_{BV}(k) = \frac{1}{c} \langle \mathbf{v} \cdot (\mathbf{J} \times \mathbf{B}) \rangle$$
$$T_{BV}(k_{hor}, z') = \frac{1}{c} \mathfrak{F}\{\mathbf{v}(x, y, z')\} \cdot \mathfrak{F}\{[\mathbf{J} \times \mathbf{B}](x, y, z')\}^*$$

(see, e.g., Mininni et al. 2005)

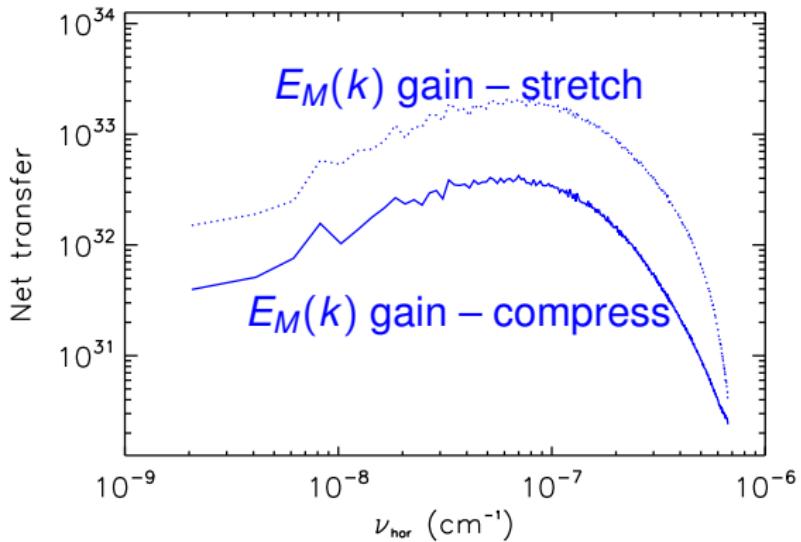
Large scales generate & small scales receive magnetic energy



At which scales is magnetic energy received?

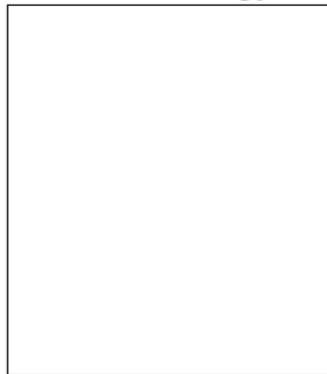


Net magnetic energy is received at all scales

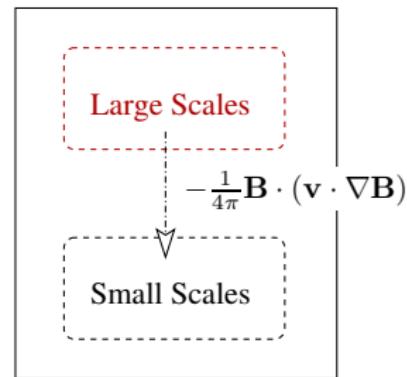


Which scales are dominated by cascade (not dynamo)?

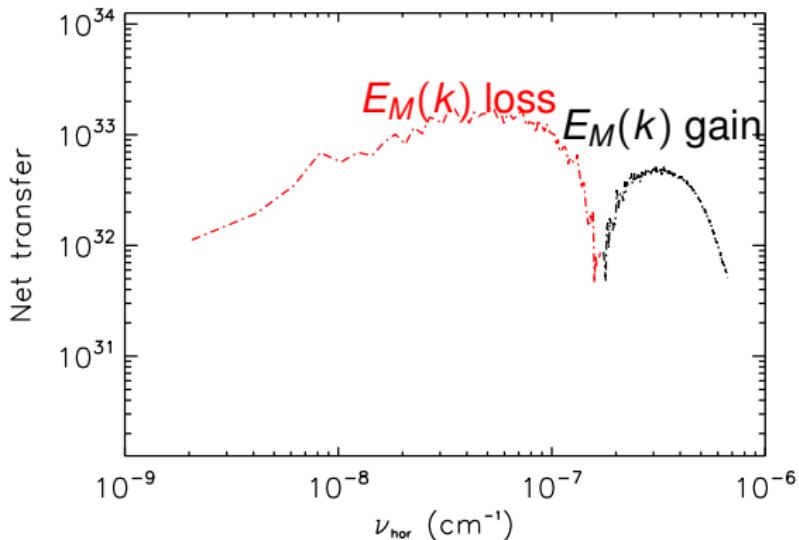
Kinetic Energy



Magnetic Energy



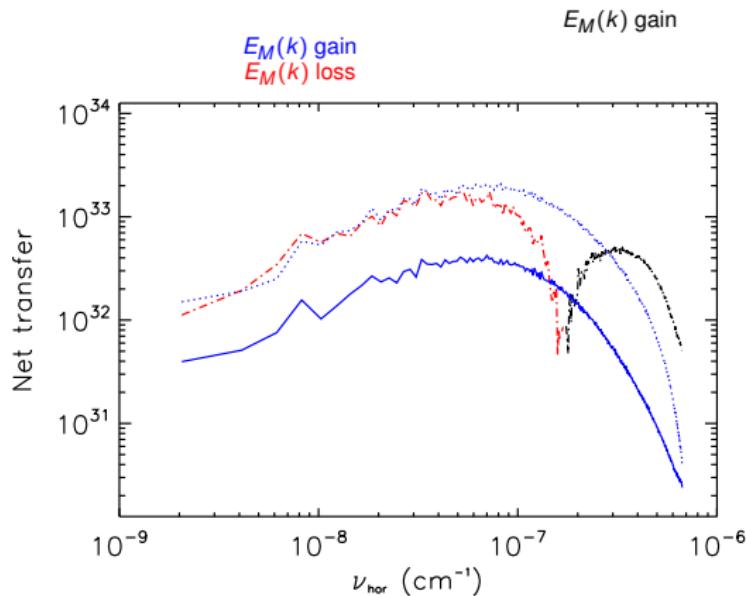
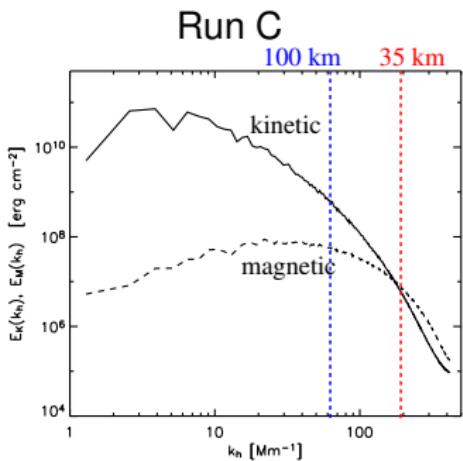
Which scales are dominated by cascade (not dynamo)?



Surface convection can support local SSD

Analysis

- 1 Cascade below ~ 35 km
- 2 \mathbf{B} generated at ~ 100 km

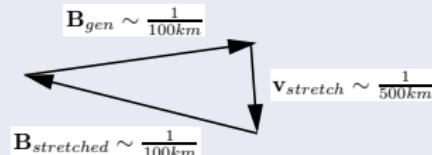


Surface convection can support local SSD

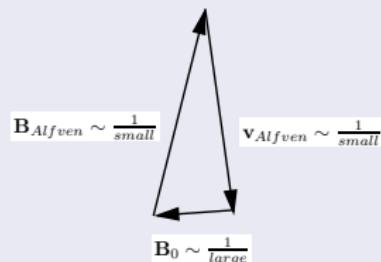
Analysis

- 1 Cascade below ~ 35 km
- 2 \mathbf{B} generated at ~ 100 km
- 3 Dynamo driven by ~ 500 km motions
- 4 Triad $\Rightarrow \mathbf{B}$ at ~ 100 km

Small-scale dynamo



Alfvénic



Triadic transfer

$$\begin{aligned}\frac{d}{dt} E_V(\mathbf{k}) &= \frac{1}{c} \hat{\mathbf{v}}_\mathbf{k} \cdot \Im\{[\mathbf{J} \times \mathbf{B}]\}_\mathbf{k}^* \\ \Im\{[\mathbf{J} \times \mathbf{B}]\}_\mathbf{k} &= \hat{\mathbf{J}}_\mathbf{p} \otimes \hat{\mathbf{B}}_\mathbf{q} \\ \Rightarrow \mathbf{k} &= \mathbf{p} + \mathbf{q}\end{aligned}$$

Conclusions

Conclusions

- Solar surface convection can support local dynamo action
- This dynamo is a small-scale dynamo
not magnetic cascade nor Alfvénization

Continuing Work

- Poynting flux
- 3D transfer (isotropy, spectral leakage, ...)
- $T_{VB}(q, k)$, $T_{BV}(q, k)$, $T_{BB}(q, k)$, ...
- Comparison with Boussinesq convective SSDs